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The Definition of Modulation and its Reduction Sentences*

La definición de modulación y sus sentencias de reducción

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Abstract

Modulation is an essential concept in the theory of mental models. According to this theory, sentences in natural language are linked to possibilities. However, the meaning of expressions and the situations in which they are used can cause changes in those possibilities. The theory calls 'modulation' to that phenomenon. Some of its defenders (Johnson-Laird, Khemlani, and Goodwin) even gave an explicit definition of it. Thus, the main aim of this paper is to address that definition in order to check if it follows or not the criteria proposed by Carnap for definitions and reduction sentences. The conclusion is that, although the definition of modulation in the theory of mental models is very abstract, it seems to fulfill those criteria.

Keywords: Definition, mental model, modulation, possibility, reduction sentence.

Resumen

Un concepto esencial en la teoría de los modelos mentales es el de modulación. Según esta teoría, las sentencias en lenguaje natural están vinculadas a posibilidades. No obstante, el significado de las expresiones y las situaciones en que son usadas pueden provocar cambios en esas posibilidades. La teoría denomina a tal fenómeno 'modulación'. Algunos de sus defensores (Johnson-Laird, Khemlani y Goodwin) ofrecieron, incluso, una definición explícita del mismo. Así, el objetivo de este trabajo es revisar dicha definición con el fin de comprobar si sigue o no los criterios propuestos por Carnap para definiciones y sentencias de reducción. La conclusión es que, aunque la definición de modulación en la teoría de los modelos mentales es muy abstracta, parece cumplir esos criterios.

Palabras clave: Definición, modelo mental, modulación, posibilidad, sentencia de reducción.

Introduction

The theory of mental models (e.g., Khemlani, Byrne, & Johnson-Laird, 2018) has claimed interesting ideas about reasoning and language. One of them, which is also one of its most important theses, is that, when processing sentences, people analyze the possibilities in which those sentences can be true (see also, e.g., Byrne & Johnson-Laird, 2020). The following example shows this:

(1) "Pat visited England or she visited Italy, or both" (Johnson-Laird, Khemlani, & Goodwin, 2015: 204).

If it is assumed that 'A' stands for the fact that Pat visited England, and 'B' represents the fact that Pat visited Italy, based on the theory of mental models, (1) refers to these three possibilities:

(2) Possible (A & B) & Possible (A & not-B) & Possible (not-A & B)

The three main conjuncts in (2) reveal all the cases in which (1) can be true. Only one scenario is impossible: the case in which Pat visited neither England nor Italy.

However, the possibilities do not depend on syntax. The connective —in (1), disjunction— does not determine them (see also, e.g., Johnson-Laird, 2010). Apparently, (3) has the same logical structure as (1). Nevertheless, its possibilities are not those in (2).

(3) "Pat visited Milan or she visited Italy" (Johnson-Laird et al., 2015: 204).

If 'A' means that Pat visited Milan now, the possibilities of (3) are just:

(4) Possible (A & B) & Possible (not-A & B)

The second possibility in (2) is removed in (4) because it is not conceivable that Pat travels to Milan without travelling to Italy.

Following the theory of mental models, this phenomenon occurs not only with disjunction. It can be observed with other connectives too (see, e.g., for the conditional, Johnson-Laird & Byrne, 2002). In this way, in the literature of cognitive science, many experimental results supporting the precedent account are to be found (see, in addition, e.g., Quelhas & Johnson-Laird, 2017; Quelhas, Johnson-Laird, & Juhos, 2010). For example, for cases akin to (1), (2), (3), and (4), two decisive tasks have been proposed to participants. In the first one, it is presented an inference with only one premise. That premise affirms a fact. On the other hand, the conclusion is a disjunction in which the fact in the premise is the right disjunct. Thereby, in a hypothetical task based on (1), the premise would be 'Pat visited Italy' and the conclusion would match (1). In this kind of task, beyond what is provided by classical logic, people mainly tend to state that the inference is not correct. The reason for that from the theory of mental models is that sentences such as (1) are related to possibilities such as the ones in (2). That causes a contradiction, since the second possibility in (2) denies the premise: it provides that Pat was in England and not in Italy (experimental conditions with this structure, their results, and explanations such as this one can be found in, e.g., Orenes & Johnson-Laird, 2012).

The second task is similar. The only difference is that the conclusion is not akin to (1), but (3). Accordingly, its possibilities are not (2), but (4). This is relevant because in these cases individuals often deem the inference as correct. The reason is not hard to understand under the theory of mental models either: now, the inconsistent possibility, the second one in (2), is not present. So, there is no incompatibility (tasks of this kind, their results, and accounts such as this last one are presented in works such as Orenes & Johnson-Laird, 2012, as well).

The name of this phenomenon is 'modulation'. The proponents of the theory of mental models have offered an explicit definition:

(5) "The process in the construction of models in which content, context, or knowledge can prevent the construction of a model and can add information to a model" (Johnson-Laird, *et al*., 2015: 202).

Obviously, 'model' in (5) is equivalent to possibility. The possibilities of sentences are models and models are 'conjunctions of possibilities' (see, e.g., in addition, Johnson-Laird & Ragni, 2019; Khemlani, Hinterecker, & Johnson-Laird, 2017). However, perhaps what is most important now is that (5) is the main point this paper will address. The aim is not to keep analyzing the concept of modulation or to give a critical review of it. As said, the pieces of evidence in the literature appear to be clear. The present paper will be intended only to explore to what extent Carnap's philosophy of science continues to be suitable to capture even such a complex and abstract definition as (5). In this regard, the first section will comment on some key issues of Carnap's framework. They will be basically his concepts of 'definition' and 'reduction sentence'. Then, the paper will try to check whether or not these last concepts fit the manner modulation is understood by the theory of mental models.

Definitions and reduction sentences in Carnap's approach

Following several works (Carnap, 1936, 1937, 1947) one might think that what Carnap considers as a definition is evident. It is an equivalence: a biconditional relation in which both clauses have to be true at the same time (or false at the same time). Thus, for example, it can be said that A is defined by B if:

(6) A is equivalent to B.

But it is possible to go any further. Based, in particular, on Carnap (1947), it could be stated that the relation should be not only of equivalence, but also of L-equivalence. According to Carnap (1947), two clauses are L-equivalent if they hold in exactly the same

state-descriptions (or possible worlds). There cannot be a particular state-description (or possible world) in which one of the clauses can be accepted and the other one cannot. From this modal logical point of view, it could be better claimed that A is defined by B if:

(7) A is L-equivalent to B.

Nevertheless, Carnap thinks that it is difficult to come to complete definitions. Reduction sentences are often necessary. This is because those sentences can progressively confirm what can be included in a particular definition. There are several types of reductions sentences. The simplest one, for instance, for R (which is deemed here as a predicate) is as follows:

(8) $(x)\,(Px \rightarrow (Qx \rightarrow Rx))$

With other symbols, (8) is sentence (R) in Carnap (1936: 442). In it, '(x)' means that x, which is a variable, is under the action of a universal quantifier. As R, P and Q are predicates. The symbol '®' denotes material conditional relation.

But, as pointed out by Carnap (1936), (8) is not always a reduction sentence for R. It is only if this formula is valid at once:

(9) $\neg(x)\,\neg(Px \wedge Qx)$

Where '¬' expresses negation and '∧' is conjunction.

In this way, what (9) indicates is that there has to be, at least, some element with properties P and Q.

Thereby, the chief point Carnap makes regarding this is that definitions can be gradually built by means of sentences with structures such as the one of (8), and, of course, (9). As far as the aim of the present paper is concerned, this can imply that, if 'Px' refers to the fact that x is a sentence, and 'Rx' to the fact that x is modulated, by attributing different properties to Q, it is possible to come to the entire definition of modulation. It will be shown that this is the case for (5) below. Nonetheless, the purpose is not to argue that Johnson-Laird

et al. (2015) took processes such as this described by Carnap (1936) into account to offer (5). Definition (5) is a current successful definition in cognitive science with empirical support. So, as mentioned, the main goal is just to review whether or not a definition of that kind can be understood under the criteria given by Carnap for the construction of scientific language. The next sections examine the different reduction sentences that could be linked to (5).

The action of content

Paying attention to (5), the essential elements playing a role in modulation appear to be three: content, context, and knowledge. This is because, according to (5), these are the three aspects explicitly mentioned that can modify possibilities. It is not hard to think about a reduction sentence for the first one (content):

(10) $(x)\,(Px \rightarrow (Q1x \rightarrow Rx))$

Where 'Q1x' stands for the fact that x has a content that is not compatible with all the usual possibilities of a sentence with its formal structure.

It is difficult to find cases refuting (10). However, it is not to propose sentences in natural language causing its progressive confirmation. If disjunction keeps being the example, one of those sentences is (11).

(11) Either you eat rice or you eat both rice and chicken.

Although (11) is a disjunction, its possibilities are not those in (2). If 'A' denotes that you eat rice and 'B' represents that you eat chicken, the conjunction of possibilities for (11) is (12).

(12) Possible (A & B) & Possible (A & not-B)

Indeed, if the two disjuncts in (11) are true, you eat both rice and chicken, which is what is indicated in the first conjunct or possibility in (12). If only the first disjunct in (11) is correct, you eat rice but not chicken, which is what the second conjunct or possibility in (12) provides. No more options are possible. On the one hand, it cannot be admitted that, in (11), the first disjunct is false and the second one true. If that were the case, there would be a contradiction: you would not eat rice (the first disjunct is false) and you would eat rice (the second disjunct is true). On the other hand, if (11) is true, its two disjuncts cannot be false at the same time. Therefore, (11) is a clear example that modulation can occur by virtue of content.

Furthermore, two more points are important here. First, modulation is not related to context or knowledge in this case. The process described would happen in any context in which (11) were expressed. Besides, knowledge does not play a role either. The mental process of modulation does not even require to know what 'rice' or 'chicken' exactly mean. The fact that 'rice' appears in the second disjunct again already guarantees modulation.

Second, this kind of modulation can occur even if the sentence includes negations. If the sentence is (13),

(13) Either you do not eat rice, or you do not eat rice or chicken.

The possibilities continue not to be those in (2). They are the following:

(14) Possible (not-A & B) & Possible (not-A & not-B)

Now, the content indicates that you can eat rice in no case, whether or not you eat chicken.

The action of context

To give a reduction sentence for context, it can enough to replace (10) with (15).

(15) $(x)\,(Px \rightarrow (Q_2x \rightarrow Rx))$

Now, 'Q_2x' represents a situation in which x is stated in a context that alters the habitual possibilities of its logical structure.

In the same way as (10), it is not easy to offer a sentence that refutes (15). However, it is simple to imagine contexts making it correct. One of them can be as follows:

Some individuals at a cafeteria have the menu. That menu allows choosing between two desserts: an apple or an orange. All of them pick the apple. But, when they finish eating, they ask the person in charge: may we have an orange as well? A negative answer in this context can have different forms. Nevertheless, keeping resorting to disjunction as an example, one of those forms could be (16).

(16) Either you eat an apple or you eat an orange.

Because of the previous context, only one possibility can be related to (16). If 'A' denotes that you eat an apple and 'B' expresses that you eat an orange, the possibility is that in (17).

(17) Possible (A & not-B)

It is not possible to eat the two fruits. The scenario in which you eat an orange and you do not eat an apple is not possible either, since you already ate an apple. In addition, if (16) is true, one of the two fruits must be eaten. Therefore, it is obvious that context can also be a predicate to consider in a reduction sentence for modulation.

Thus, the action of context is evident. Without the context indicated, the content of (16) would lead to the possibilities in (2). On the other hand, people's knowledge about apples and oranges does not have an influence here either. The fact that people know what an apple and an orange are does not modulate the possibilities.

Furthermore, the example can also include a negation in this case. Given the same context, the answer could also be (18).

(18) Either you do not eat an apple or you do not eat an orange.

Although (18) is a disjunction too, its possibility continues to be (17). You ate an apple and you cannot eat an orange.

The action of knowledge

Lastly, the reduction sentence for knowledge could be (19).

(19) $(x)\,(Px \rightarrow (Q_3x \rightarrow Rx))$

In this case, 'Q_3x' refers to a circumstance in which people's general knowledge can change the regular possibilities corresponding to the syntactic structure of x.

Once again, an instance against (19) is hard to propose, and one supporting that very sentence is not. If disjunction continues to be the example to consider, a sentence confirming (19) can be (3). As explained, its possibilities are those in (4). Hence, (4) shows in an evident way that individuals' knowledge (in this case, their geography knowledge) can have an influence on modulation too.

The action of knowledge is different from the action of content. That is not hard to note with the examples above. In (3) the content of the first disjunct does not appear, as in (11), in the second disjunct again. Likewise, context is not involved in (3) either. At least currently, Milan is a city in Italy.

Lastly, an example with a negation related to knowledge is possible as well. That can be (20).

(20) Either Pat visited Milan or she did not visit Italy.

Coming back to the equivalences for 'A' and 'B' linked to Milan and Italy above, (20) continues not to refer to (2). (20) only allows one possibility: (21).

(21) Possible (A & B)

Accordingly, the three essential elements that can act in modulation can be captured by means of reduction sentences. Sentences (10), (15), and (19) are confirmable, and that fact is very relevant to the general phenomenon of modulation. Nevertheless, several more points deserve to be taken into account.

On the one hand, it is really difficult to give more factors playing a role in modulation. For this reason, it can be assumed that (10), (15), and (19) together describe all the properties that can be assigned to it. So, a definition of modulation with the structure claimed by Carnap and, in addition, matching (5) can be built. That definition could be, for instance, (22).

(22) Rx is L-equivalent to $Q_1x \vee Q_2x \vee Q_3x$.

Where '$\vee$' denotes disjunction.

In this way, it seems possible to claim that the thesis of modulation in the theory of mental models satisfies Carnap's criterion expressed by means of (8) and (9) in two senses. First, given that the theory only proposes content, context, and knowledge as factors of modulation, formulae (10), (15), (19), and (22) appear to show something similar to logical correspondence between the thesis and Carnap's requirement. Second, it is also possible to speak about correspondence by empirical application. This last idea can be argued in at least two senses too. It is difficult to find more factors having an influence on modulation. In addition, the experiments in the literature on the theory of mental models (e.g., Johnson-Laird & Byrne, 2002; Orenes & Johnson-Laird, 2012; Quelhas & Johnson-Laird, 2017; Quelhas, *et al.*, 2010) seem to confirm that the three mentioned factors, content, context, and knowledge, cause modulation.

On the other hand, another aspect to insist in is that the present paper has focused only on the example of disjunction. As mentioned, the literature also reveals modulation processes in the cases of other connectives (a connective that has been very studied in this regard is especially the conditional; see, e.g., Orenes & Johnson-Laird, 2012).

Besides, examples with negations have been proposed above as well. Therefore, it is evident that (10), (15), (19), and (22) could be applied to those connectives as well.

Thus, the possibilities for research this paper opens are varied. It is possible to continue to analyze definitions of the theory of mental models from the requirements established by Carnap. This would allow checking whether those requirements are fulfilled with other definitions of the theory too. Furthermore, it is also possible to work in the same direction in the general field of cognitive science, presenting studies with no restriction to just the theory of mental models. The benefits would be obvious. It could be analyzed whether the definitions in the theories can be expressed by resorting to reduction sentences or not. This in turn would enable to verify the scope the theories try to have and the aspects of reality they truly address.

In fact, following Carnap's intentions, the review could be extended to other scientific disciplines as well, not taking only cognitive science into account. One point would be that the assessment of the definitions selected would enable to see whether a framework such as the one of Carnap keeps being applicable nowadays. Regarding this, it can be said that there are already papers that have claimed the idea of retrieving some aspects of Carnap's thought to use them at present (e.g., López-Astorga, 2019). However, another point would be that the use of reduction sentences would also allow determining the real extent, perspectives, and objects the theories deal with in disciplines other than cognitive science.

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