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Physical obligation, Modus Tollendo Tollens, and the Stoic criterion*

Obligación física, Modus Tollendo Tollens y el criterio estoico

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Abstract

The literature in the field of cognitive science reports that people only apply Modus Tollendo Tollens in some cases. A representative case in which individuals apply that logical schema is when the conditional premise consists of a physical obligation. In this paper, I try to show that a non-axiomatic logic can make inferences of Modus Tollendo Tollens just in the cases like that. The key is that the conditional is coherent with the Stoic criterion. That allows considering the negation of the consequent and the negation of the antecedent as clauses in a same implication statement with the highest frequency.

Keywords

Conditional, Modus Tollendo Tollens, Non-Axiomatic Logic, physical obligation, Stoic criterion.

Resumen

La literatura en el área de la ciencia cognitiva informa que las personas solo aplican Modus Tollendo Tollens en algunos casos. Un caso representativo en el que los individuos aplican tal esquema lógico es cuando la premisa condicional consiste en una obligación física. En este trabajo, trato de mostrar que una lógica no axiomática puede realizar inferencias de Modus Tollendo Tollens solo en situaciones similares a esa. La clave reside en que el condicional sea coherente con el criterio estoico. Ello permite considerar la negación del consecuente y la negación del antecedente como cláusulas en una misma afirmación de implicación con la frecuencia más alta.

Palabras clave

Condicional, criterio estoico, Lógica No-Axiomática, Modus Tollendo Tollens, obligación física.

Introduction

Let us suppose that we know that the following statement is true:

“If the patient has malaria then she has a fever” (Johnson-Laird & Byrne, 2002: 663; Table 4).

If we also know that a particular patient has malaria, we can deduce that she has a fever. The premises would be:

If the patient has malaria, then she has a fever.
The patient has malaria.

And the conclusion would be:

The patient has a fever.

This inference has the structure of *Modus Ponendo Ponens*. It was apparently proposed by Chrysippus of Soli (Diogenes Laertius, *Vitae Philosophorum*, 7, 80). Experimental studies reveal that most people make inferences of this kind with no difficulties (Braine & O’Brien, 1998).

The situation would be different if the second premise matched the negation of the consequent of the conditional, that is, if it were:

The patient does not have a fever.

In this case, the conclusion would be:

The patient does not have malaria.

The latter inference has the logical form of *Modus Tollendo Tollens*. It is also habitually attributed to Chrysippus of Soli (Diogenes

Laertius, *Vitae Philosophorum*, 7, 80). However, experimental studies indicate that this inference is harder: to come to its conclusion is more difficult (Byrne & Johnson-Laird, 2009).

But there are circumstances in which we can expect a good performance in reasoning tasks with the structure of Modus Tollendo Tollens. For example, an idea is that, when the conditional in the first premise is an obligation, the number of individuals responding correctly increases (Byrne, 2005; see also Cramer, Hölldobler, & Ragni, 2021). Another idea is that, when the conditional is coherent with the Stoic criterion, that is, the criterion Chrysippus provided (e.g., Sextus Empiricus, *Pyrrhoniae Hypotyposes*, 2, 111), to make an inference of Modus Tollendo Tollens should be easier (López-Astorga, 2015). I will focus on the latter idea.

It is well known that the Stoic criterion establishes that an incompatibility should exist in the conditional (when it is true): the negation of the consequent should be inconsistent with the antecedent (see also Cicero, *De Fato*, 12). When this happens, the individual knows that if the consequent is negated, the antecedent cannot be true (see also Gould, 1970). Building on the example above, the individual knows that

If the patient does not have a fever, then the patient does not have malaria.

Given the latter conditional and the second premise in the previous Modus Tollendo Tollens inference, that is, ‘the patient does not have a fever’, the inference is transformed into a Modus Ponendo Ponens inference, whose conclusion is ‘the patient does not have malaria’ (for a detailed explanation of this argument, see, e.g., López-Astorga, 2015).

What I try to show in this paper is that there is a computer system that can work in this way, that is, making Modus Tollendo Tollens inferences only when the Stoic criterion is fulfilled. The name of the system is Non-Axiomatic Reasoning System (NARS), and has been developed in several works (e.g., Wang, 2006). Its underlying logic has a name, too: Non-Axiomatic Logic (NAL), which has been described

in different works as well (e.g., Wang, 2013). I will explain below why, while NAL generally cannot make Modus Tollendo Tollens inferences, it can make them when Chrysippus' criterion is followed.

There are already works linking the Stoic criterion to NAL. For example, it has been argued that, when the conditional stands for a deontic obligation and is coherent with Chrysippus' criterion, an idealized version of NAL allows making inferences at least compatible with the expected conclusion in Modus Tollendo Tollens (López-Astorga, 2025).

I will not deal with deontic obligations here, but non-deontic statements fulfilling the Stoic criterion. Those statements can be called 'physical obligations'. 'Obligation' in the latter expression does not have its habitual meaning in fields such as ethics or deontic logic. The expression is present in the literature indicating a physical relation between the contents of the clauses of a conditional: if the first one occurs, the second one must be the case as well (Cramer *et al.*, 2021). I think that the existence of that relation implies that the conditional satisfies Chrysippus' criterion. So, I will use 'physical obligation' in the present paper to refer to non-deontic conditionals that are coherent with that criterion. My example will be that indicated above (i.e., 'if the patient has malaria, then the patient has a fever'). Besides, I will consider NAL, not an idealized version of it.

The paper will have three sections. The two first ones will be devoted to essential components of NAL. The second one will be particularly important. It will show why Modus Tollendo Tollens cannot be usually applied in NAL. The third section will indicate why, despite what was revealed in the second section, Modus Tollendo Tollens inferences are possible in NAL when the Stoic criterion is considered.

Implication and inheritance

'Narsese' is the name of the language of NARS (Wang, 2013). The statement 'if a patient has malaria, then the patient has a fever' can be expressed in Narsese as follows:

$$(1) (\#X \rightarrow M) \Rightarrow (\#X \rightarrow F) \langle f, c \rangle$$

Where ‘ M ’ and ‘ F ’ represent, respectively, ‘to have malaria’ and ‘to have a fever’.

The present section will deal with symbols ‘ \Rightarrow ’ and ‘ \rightarrow ’, which stand for, respectively, the ‘implication copula’ and the ‘inheritance copula’ (see also Wang, 2023).

Implication (‘ \Rightarrow ’) in NAL is different from what it is in first-order predicate calculus, that is, in classical logic. In principle, it seems to be similar, as in NAL

$$(X \Rightarrow Y) \text{ IFF } (\{X\} \Vdash Y)$$

(I am using ‘IFF’ at the metalevel; it is common to use first-order predicate logic as a metalanguage to explain NAL; see Wang, 2013).

That is, $X \Rightarrow Y$ means that, with a finite number of steps, the application of rules existing in the system leads from X to Y (Wang, 2013).

But NAL has an essential assumption (which is also described in Wang, 2011): the ‘Assumption of Insufficient Knowledge and Resources’ (AIKR). The system can never be sure about the truth of a statement. This is the reason why NAL is a non-axiomatic logic: nothing is ever completely true within it. The experience the system acquires modifies the truth values of the statements. This is one of the main characteristics of the logic, as it causes the system to seem to reason as a human being (Wang, 2013). All the statements in NAL have a ‘frequency value’ (f). All of them have a ‘confidence value’ (c), too. They are present in (1) on the right. The pieces of evidence the system has allow calculating them (Wang, 2013; Definition 3.3).

In a statement such as $X \Rightarrow Y$, the first clause, that is, X , is a sufficient condition for the second clause, that is, Y (the fact that X is the case is sufficient for Y to also be the case), and the second clause, that is, Y , is a necessary condition for the first clause, that is, X (when Y is the case, X must be necessarily the case, too). X can have both its sufficient conditions and other necessary conditions. Y can have

both other sufficient conditions and its necessary conditions (Wang, 2013; Definition 9.3).

Let X^S , X^N , Y^S , and Y^N be, respectively, the set of sufficient conditions of X , the set of necessary conditions of X , the set of sufficient conditions of Y , and the set of necessary conditions of Y . Let w^+ , w^- , and w be, respectively, the set of pieces of ‘positive evidence’ in the system, the set of pieces of ‘negative evidence’ in the system, and the set of pieces of ‘total evidence’ in the system. They are calculated in this way (see Wang, 2013, Definitions 3.2 and 9.4):

$$w^+ = (X^S \cap Y^S) + (X^N \cap Y^N)$$

$$w^- = (X^S - Y^S) + (Y^N - X^N)$$

$$w = (X^S \cap Y^S) + (X^N \cap Y^N) + (X^S - Y^S) + (Y^N - X^N)$$

So, w^+ includes the sufficient and necessary conditions X and Y share, w^- includes the sufficient conditions of X that are not sufficient conditions of Y and the necessary conditions of Y that are not necessary conditions of X , and w includes the sum of w^+ and w^- .

The system can be always learning and acquiring knowledge. However, because of AIKR, its information is limited every time (Wang, 2011, 2013). Let us suppose that, by virtue of AIKR, in a particular moment, the system only knows these statements about X and Y :

$$\{A \Rightarrow X, B \Rightarrow X, C \Rightarrow X, A \Rightarrow Y, B \Rightarrow Y, Y \Rightarrow D, Y \Rightarrow E, Y \Rightarrow F, X \Rightarrow D, X \Rightarrow E\}$$

If this set is considered, we have that

$$X^S = \{A, B, C\}$$

$$Y^S = \{A, B\}$$

$$X^N = \{D, E\}$$

$$Y^N = \{D, E, F\}$$

So, for $X \Rightarrow Y$,

$$w^+ = \{A, B\} + \{D, E\} = 4$$

$$w^- = \{C\} + \{F\} = 2$$

$$w = 4 + 2 = 6$$

Based on this, to calculate f and c is easy: “ $f = w^+/w$ ” (Wang, 2013: 29; Definition 3.3); “ $c = w^-(w^+ + w^-)$ ” (Wang, 2013: 29; Definition 3.3). In the formula of c , a constant appears: k . Its usual value is 1. A higher value can make the system too distrustful (for a discussion, see Wang, 2013).

Therefore, the result is $f = 4/6 = 0.67$; $c = 6/7 = 0.86$. The correct way to represent $X \Rightarrow Y$ in this case is:

$$X \Rightarrow Y \langle 0.67, 0.86 \rangle$$

Calculations of this kind are necessary by virtue of AIKR. New information and new evidence can update and change the values of f and c in any time (Wang, 2013).

Within both the antecedent and the consequent in (1), there is another symbol: ‘ \rightarrow ’. It is the ‘inheritance copula’ (Wang, 2013, 2023). Let us ignore symbol ‘#’ for the moment. What ‘ $X \rightarrow M$ ’ means is that X is in the extension of M (i.e., X is a particular example of M), and M is in the intension of X (i.e., M is a general property of X). Likewise, what ‘ $X \rightarrow F$ ’ stands for is that X is in the extension of F (i.e., X is a particular example of F), and F is in the intension of X (i.e., F is a general property of X) (see Wang, 2013; Definition 2.8). Beyond this, extension and intension are not understood in NAL as habitual in logic literature (see also Wang, 2023).

The system proposes a ‘partial isomorphism’ between ‘ \Rightarrow ’ and ‘ \rightarrow ’ (Wang, 2013). This means that the statements with ‘ \rightarrow ’ can also have values for f and c . The same formulae indicated above can be used. It is only required to change the sets of sufficient conditions for the sets of elements in extensions, and the sets of necessary

conditions for the sets in intensions (Wang, 2013; Definitions 2.8, 3.2, 3.3, 9.3, and 9.4; Theorems 2.4 and 9.2).

However, I will not consider values of f and c for inheritance statements in the present paper. The reason is symbol ‘#’. That symbol provides that the variable following it – in the case of (1), X – is an independent variable. Independent variables in NAL can act as the universally quantified variables in first-order predicate calculus (Wang, 2013; Definition 10.2), that is, variables providing that what applies to them applies to all the element considered, too. Because of this, my argumentation below can be developed without resorting to the values of f and c for the inheritance statements in the antecedents and the consequents of implications.

Deduction and contraposition

Let us suppose that ‘T’ means ‘to be tested positive for malaria’. If the system includes (1) and (2),

$$(2) (\#X \rightarrow T) \Rightarrow (\#X \rightarrow M) \langle f_2, c_2 \rangle$$

It can derive (3) by virtue of one of its rules: deduction (Wang, 2013).

$$(3) (\#X \rightarrow T) \Rightarrow (\#X \rightarrow F) \langle f_3, c_3 \rangle$$

The values of f_3 and c_3 depend on those of f , c , f_2 , and c_2 . They are calculated in this way: $f_3 = f \times f_2$; $c_3 = f \times c \times f_2 \times c_2$ (Wang, 2013; Table 4.7).

Let us assume the following values for f , c , f_2 , c_2 :

$$(\#X \rightarrow M) \Rightarrow (\#X \rightarrow F) \langle 1, 0.9 \rangle$$

$$(\#X \rightarrow T) \Rightarrow (\#X \rightarrow M) \langle 1, 0.9 \rangle$$

I have taken those values because they are high enough. Besides, they are the values the system assigns to f and c when the frequency and the confidence are not indicated (Wang, 2013). With them, the values of the conclusion would be:

$$(\#X \rightarrow T) \Rightarrow (\#X \rightarrow F) \langle 1, 0.81 \rangle$$

Given that the inheritance statements can also have values of f and c , it is possible to make deductions of this type with them (Wang, 2013). However, it is not necessary to consider this point to present my argumentation.

This logical context allows Modus Ponendo Ponens inferences. If a statement is in the system, there is evidence in the system supporting it. Thus, we can assume (as in Wang, 2013), that $\{E\}$ is the set of the statements in the system leading to a particular statement. For example, let us think of the case of $(\#X \rightarrow M)$. We can infer $(\#X \rightarrow F)$ from (1) and $(\#X \rightarrow M)$ because we know that there is at least a set $\{E\}$ such that

$$(4) E \Rightarrow (\#X \rightarrow M) \langle f_4, c_4 \rangle$$

$\{E\}$ is not the only possible set of pieces of evidence in favor of $(\#X \rightarrow M)$. The same formula can be supported by different sets of pieces of evidence at the same time, and with different values of f and c for each of those sets (this is because of AIKR; Wang, 2013). But, as said, if a statement is in the system, there is at least a set $\{E\}$ from which it derives.

From (1) and (4), we can obtain (5).

$$(5) E \Rightarrow (\#X \rightarrow F) \langle f_5, c_5 \rangle$$

Where $f_5 = f \times f_4$; $c_5 = f \times c \times f_4 \times c_4$.

The point is that (5) can be simplified as (6).

$$(6) (\#X \rightarrow F) \langle f_5, c_5 \rangle$$

This is Modus Ponendo Ponens. From (1) and $(\#X \rightarrow M)$, that is, (4), we have inferred (6) (for a similar explanation of this process, see, Wang, 2013).

Nevertheless, the same cannot be said about Modus Tollendo Tollens. Modus Tollendo Tollens includes negations. The values of f and c for the negation of a statement in NAL can be calculated by considering the value of w^+ as the value of w^- , and the value of w^- as the value of w^+ (Wang, 2013; Definition 9.7).

In the case of the ‘contraposition rule’ (i.e., the rule allowing replacing a conditional with another in which antecedent is the negation of its consequent, and the consequent is the negation of its antecedent), we have this formula:

$$“w = w^- = (1 - f_1) \times c_1” \text{ (Wang, 2013: 130; Table 9.8).}$$

Where f_1 and c_1 are the values of frequency and confidence of the premise.

If we have this formula:

$$X \Rightarrow Y \langle 1, 0.9 \rangle$$

We can derive this one from it.

$$\neg Y \Rightarrow \neg X \langle 0, 0 \rangle$$

Given that $w = w^-$, $w^+ = 0$ will be always the case. So, in the conclusion of the contraposition rule, $f = 0$ will be always the case (see Wang, 2013, for a detailed explanation). In this scenario, it is hard to use Modus Tollendo Tollens.

However, we can argue that, when the implication copula in NAL fulfills the Stoic criterion, this can be different. As said, Chrysippus' criterion has already been considered in works assuming NAL. But the link in those works have been provided by means of the inheritance copula, not the implication copula (López-Astorga, 2025).

It has been said that, when in an inheritance statement $f = 1$, that statement fulfills the Stoic criterion. This relation is based on two points. On the one hand, following works such as that of Lenzen (2019), we can think that the inclusion criterion, that is, the criterion claiming that the first clause in a conditional includes the consequent, is not different from the Stoic criterion. This brings the Stoic criterion to inheritance relations, since the second term is in the intension of the first term in the latter relations. On the other hand, the Stoic criterion means that it is not possible that the consequent of the conditional is false in a scenario in which the antecedent is true (see also Sedley, 1984). This idea leads to deem a conditional following Chrysippus' criterion as an inheritance statement with $f = 1$ (as pointed out, these two points are developed in works such as López-Astorga, 2025).

Given the isomorphism indicated above, it is not hard to move from the inheritance copula to the implication copula. In fact, one might think that to link the Stoic conditional to a copula representing implication is more appropriate than to link it to a copula representing inheritance. After all, it is easy to assume that the necessary conditions somehow derive from the concept of the antecedent in every implication in NAL. Besides, the need for $f = 1$ can keep being held with no difficulties.

What should be added to the literature relating Chrysippus' conditional to NAL is that something must guarantee that the negation of the consequent is impossible in a context with the antecedent being true. The system needs to know, for example, that

$$(X \Rightarrow Y \langle f_6, c_6 \rangle) \text{ IFF } (\neg Y \Rightarrow \neg X \langle f_7, c_7 \rangle)$$

Where $f_6 = f_7 = 1$, and the values of c_6 and c_7 are high enough. Otherwise, $X \Rightarrow Y \langle f_6, c_6 \rangle$ does not fulfill the Stoic criterion.

As shown in the previous section, it is not possible to infer, with $f \neq 0$, $\neg Y \Rightarrow \neg X \langle f_7, c_7 \rangle$ from $X \Rightarrow Y \langle f_6, c_6 \rangle$. While $f_6 = 1$ could be the case, $f_6 \neq f_7$ and $f_7 = 0$ would hold, too. However, this problem can be solved.

The assumption of NAL is AIKR. It is a system that can always update its values of f and c by virtue of experience. In addition, the system can have the same formula with different values of f and c at the same time. The reason is that the formula can come from different sets of pieces of evidence. Let us imagine two sets of pieces of evidence: $\{E1\}$ and $\{E2\}$. (7) and (8) can coexist in the system.

$$(7) \quad \neg Y \Rightarrow \neg X \langle 0, 0 \rangle$$

$$(8) \quad \neg Y \Rightarrow \neg X \langle 1, 0.9 \rangle$$

This is because (7) can be derived from $E1 \Rightarrow (X \Rightarrow Y) \langle 1, 0.9 \rangle$, via $X \Rightarrow Y \langle 1, 0.9 \rangle$ and the contraposition rule. On the other hand, (8) can be a simplified form of $E2 \Rightarrow (\neg Y \Rightarrow \neg X) \langle 1, 0.9 \rangle$.

Which would the option of the system be? (7) or (8)? Within NAL, there are several possibilities. If the intersection of $\{E_1\}$ and $\{E_2\}$ is not the empty set, that is, if $\{E_1\} \cap \{E_2\} \neq \emptyset$, (set theory is also a common metalanguage to present NAL; Wang, 2013), that is, if $\{E_1\}$ and $\{E_2\}$ share elements, the system will choose the statement with higher value assigned to c (Wang, 2013): in this case, (8). If, on the contrary, $\{E_1\} \cap \{E_2\} = \emptyset$, the system will consider a new set $\{E_3\}$, which should be equivalent to $\{E_1\} \cup \{E_2\}$. Based on $\{E_3\}$, it will calculate f and c again to give new values to the statement, that is, in this case, $\neg Y \Rightarrow \neg X$ (see Wang, 2013).

Beyond this, one might think that if the system already knows (8) (and it should know it so that $X \Rightarrow Y \langle 1, 0.9 \rangle$ fulfills the Stoic criterion), it will not need to infer $\neg Y \Rightarrow \neg X \langle 0, 0 \rangle$ from $X \Rightarrow Y \langle 1, 0.9 \rangle$. But if, despite that, (7) were derived, it could be ignored because of its low values of f and c (see Wang, 2013).

So, given a physical obligation (in the sense indicated in works such as that of Carmer *et al.*, 2021) such as (1), we can expect that, if taken as the conditional premise, it allows making Modus Tollendo Tollens inferences. We can think that it is consistent with the Stoic criterion. Cases of malaria without a fever are not impossible, but they are uncommon. Hence, we can attribute to it these values of f and c :

$$(9) (\#X \rightarrow M) \Rightarrow (\#X \rightarrow F) \langle 1, 0.9 \rangle$$

Because it fulfills Chrysippus' criterion, we must assume that (10) is in the system as well.

$$(10) \neg(\#X \rightarrow F) \Rightarrow \neg(\#X \rightarrow M) \langle 1, 0.9 \rangle$$

If, for example, we have the information in (11),

$$(11) \neg(\#X \rightarrow F) \langle 1, 0.9 \rangle$$

There is at least a set $\{E\}$ such that $\{E\} \neg(\#X \rightarrow F) \langle 1, 0.9 \rangle$. As explained above, from (10), this leads to $\{E\} \Rightarrow \neg(\#X \rightarrow M) \langle 1, 0.81 \rangle$, which can be expressed as (12).

$$(12) \neg(\#X \rightarrow M) \langle 1, 0.81 \rangle$$

If we think just of (9), (11), and (12), this is an application of Modus Tollendo Tollens like that described in the introduction. It is an apparent application. During the process, it is transformed into a Modus Ponendo Ponens inference. The key is that, despite the status of the contraposition rule in NAL, statements such as (9) and (10) can coexist in that system.

Conclusions

The literature shows that Modus Ponendo Ponens is much easier to apply than Modus Tollendo Tollens. The latter schema is only applied

in particular occasions. Several explanations have been given about the reasons for this. I have focused on the idea that Modus Tollendo Tollens is applied when its conditional follows the Stoic criterion. I have tried to argue that, within a logical system such as NAL, when that criterion is fulfilled, it is possible to come to the conclusion expected in Modus Tollendo Tollens without difficulties.

Relations between NAL and the Stoic criterion are to be found in several works. It has been claimed that a conditional coherent with that criterion can be translated into a Narsese statement linking the clauses by means of an inheritance relation with $f = 1$. Here, I have analyzed the relation in depth and led it from the inheritance copula to the implication copula (which seems more appropriate for a Stoic conditional sentence). I have also added one more requirement: the existence of other implication between the negation of the consequent (which is transformed into the antecedent) and the negation of the antecedent (which is transformed into the consequent).

This allowed solving the problem of the contraposition rule in NAL. The latter rule always leads to a conclusion with $f = 0$. However, the additional requirement I have proposed enables to admit a statement such as the conclusion in that rule with $f = 1$, provided that the Stoic criterion is fulfilled. Thus, Modus Tollendo Tollens inferences can be made, since their underlying form is that of Modus Ponendo Ponens. But the point is that we can have a conditional and the negation of its consequent as premises, and, as in Modus Tollendo Tollens, the negation of the antecedent as conclusion.

The example I have used is that of a statement expressing what is called a ‘physical obligation’ in the literature. Given that different experimental studies report that Modus Tollendo Tollens tends to be applied when the conditional is a physical obligation of that kind (see Cramer *et al.*, 2021), what the present paper shows is that, at least in this regard, NAL can come to the same conclusions as human beings. Further work should review whether there is also consistency with the human responses in more Modus Tollendo Tollens tasks with different types of conditionals.

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